Preparing to succeed in

A-level Maths

College Preparation Work

Name:

Welcome to college and to the Mathematics Department.

You will now have a long break from school and may well find that you get rather rusty at some of the maths skills which you spent so long learning at school.

This booklet contains some of those key ideas from GCSE which will help you to make a good start on the A-level course. Please work through this booklet over the next few weeks to keep your skills up to speed.

The step up from GCSE to A-Level is a big leap. There are additional resources available to practice essential maths skills on the "summer homework" section of our college website https://www.scg.ac.uk/pre-enrol – please do these for extra practice.

Read through the examples for each topic and have a go at the questions in each bold section. Please make a good attempt at every question - we'd rather it was wrong than blank as it helps us to see where you may need some help! It's fine to look things up in your old books, or look at websites like BBC GCSE Bitesize to get some help if you need it.

Please set out all your working carefully and **<u>try not to use a calculator</u>** for any of these questions – algebraic skills are tested in the exams.

Preparing for lessons in September – please bring:

- A4 file paper (lined not squared is preferable)
- A ring binder folder with some file dividers
- Pens and pencils
- Highlighter pens
- This booklet to hand in!

Algebra

Substituting values, expanding brackets & collecting terms

Substitution:

If $a = 2$, $b = -3$, and $c = 5$ find the values of the following expressions:

$a^2 + b^2 =$

$3ab - 2ac =$

$(2a - b)(b + c) =$

$2c^2 - abc =$

Expanding & Simplifying Examples

Take care with ± signs

$$2(a+b) - 3(a-2b) = 2a + 2b - 3a + 6b = -a + 8b$$

$$5 - 3(a-2) = 5 - 3a + 6 = 11 - 3a$$

Multiplication before subtraction (BIDMAS)

Remember to multiply every term in the first bracket with every term in the second! [Sometimes known as FOIL]

$$(2a+3b)(a-2b) = 2a^2 - 4ab + 3ab - 6b^2 = 2a^2 - ab - 6b^2$$

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Expanding & Simplifying Examples

Try the following:

$3(a+10)+2(a-3)=$

$12-7(x+3)=$

$(3x-y)(2x+4y)=$

$(2a-3)(2a+3)=$

$(2x-3)(x-2)=$

Algebra - Solving and Rearranging Equations

Examples:

$$\mathbf{3x + 7 = 19}$$

Subtract 7 from each side

$$\Rightarrow \mathbf{3x = 12}$$

Divide each side by 3.

$$\Rightarrow \mathbf{x = 4}$$

You don't need to say what you're doing each time – this is just here to remind you!

Notice that the working is set out with one line below another.

Don't write things like this:

$3x + 7 = 19 = 3x = 12 = 4$

You may finish in the right place but this lengthy list of = signs doesn't make sense!

Solve to find the value of a:

$$7a - 2 = 13 - 2a$$
$$\Rightarrow 7a = 15 - 2a$$
$$\Rightarrow 9a = 15$$
$$\Rightarrow a = \frac{15}{9} = \frac{5}{3}$$

We prefer fractions – don't change to decimals

Rearrange to make c the subject:

$$D = b^2 - 4ac$$
$$\Rightarrow D + 4ac = b^2$$
$$\Rightarrow 4ac = b^2 - D$$
$$\Rightarrow c = \frac{b^2 - D}{4a}$$

Rearrange to make b the subject:

$$D = b^2 - 4ac$$
$$\Rightarrow D + 4ac = b^2$$
$$\Rightarrow b = \pm\sqrt{D + 4ac}$$

Remember to use ± for square roots

Solve the following:

$4x - 11 = 5$	$5 - 7z = -9$
$6 + 4(y - 1) = 12y + 2$	$5a - 3 = 11 - 2a$

$\frac{a}{2} + 10 = 3a + 2$	$(x-3)(x-2) = (x+1)(x-5)$

Rearrange the following formulae:

Make y the subject:	
$3x - y = 5$	$3y - 6x - 4 = 0$
Make a the subject:	
$b = \frac{a^2}{2}$	$b = \frac{\sqrt{a+2}}{c}$
Make r the subject:	
$S = 4\pi r^2$	$x = m - 3r$
[S is the formula for the surface area of a sphere]	

Algebra – Factorising

Examples:

Look for common factors (numbers or letters) to take out first…

$4a - 8b = 4(a - 2b)$ $x^2 - 5x = x(x - 5)$ $6yz - 9z^2 = 3z(2y - 3z)$

Quadratic expressions – look for numbers which multiply to make the constant term and add up to give the x term.

$x^2 - 5x + 6 = (x - 2)(x - 3)$

Add to make -5

Multiply to make +6

So choose -2 and -3 for the factors

$a^2 + 6a - 27 = (a + 9)(a - 3)$

Add to make +6

Multiply to make -27

So choose +9 and -3 for the factors

Sometimes do both…

$2x^3 - 14x^2 + 24x = 2x(x^2 - 7x + 12) = 2x(x - 3)(x - 4)$

Common factors first

Then factorise the quadratic

Factorise the following:

$25a + 10b =$	$x^2 + x - 12 =$
$12ab - 8bc =$	$y^2 - 11y + 30 =$
$3x^3 + 9x^2 =$	$3a^2 - 3a - 60 =$

Algebraic Fractions

Rules :
You can add or subtract fractions if they have a common denominator.
Multiply by multiplying the numerators and multiplying the denominators.
Dividing by a fraction is the same as multiplying by the reciprocal (turn it over!)
Don't forget to cancel fractions into their simplest possible form even when algebraic.
If unsure try a numerical version and use the same method for algebra.

Examples:

$$\frac{1}{x}+\frac{2}{y}=\frac{y}{xy}+\frac{2x}{xy}=\frac{y+2x}{xy}$$

$$\frac{3x^2-15x}{10}\div\frac{x-5}{5}=\frac{3x(x-5)}{10}\times\frac{5}{x-5}$$
$$=\frac{15x(x-5)}{10(x-5)}=\frac{3x}{2}$$

$$\frac{x}{8}-\frac{3}{y}=\frac{xy}{8y}-\frac{24}{8y}=\frac{xy-24}{8y}$$

$$\frac{3x}{5x^3}\times\frac{15x^5}{12x^2}=\frac{45x^6}{60x^5}=\frac{3x}{4}$$

$$\frac{1}{a}+\frac{1}{b}=$$

(Remember to simplify!)

$$\frac{3}{x^2}-\frac{1}{x}=$$

$$\frac{2}{x^3}\times\frac{x^2}{8}=$$

$$\frac{1}{x}\div\frac{1}{y}=$$

(Remember to simplify by factorising as much as possible)

$$\frac{a}{b} - \frac{a}{b+1} =$$

$$\frac{1}{x+1} + \frac{1}{x-1} =$$

$$\frac{x-4}{x+3} \div \frac{2x-8}{3} =$$

$$\frac{3a^2 - 3a - 60}{a^2 - 2a - 24} =$$

Solving Quadratics

You can solve quadratic equations in the form $ax^2 + bx + c = 0$ by

- Factorising

$$6x^2 - 11x - 10 = 0$$
$$\Rightarrow (3x+2)(2x-5) = 0$$
$$\Rightarrow 3x + 2 = 0 \text{ or } 2x - 5 = 0$$
$$\therefore \; x = -\frac{2}{3} \text{ or } x = \frac{5}{2}$$

Solve by factorising (solve means find the values of x)

$$4x^2 + 25x - 21 = 0$$

- Completing the square

$$x^2 + 12x + 3 = 0$$
$$\Rightarrow (x+6)^2 - 36 + 3 = 0$$
$$\Rightarrow (x+6)^2 - 33 = 0$$
$$\Rightarrow (x+6)^2 = 33$$
$$\Rightarrow x + 6 = \pm\sqrt{33}$$
$$\Rightarrow x = -6 \pm \sqrt{33}$$
$$\therefore \ x = -6 - \sqrt{33} \text{ or } x = -6 + \sqrt{33} \text{ [exact solutions]}$$
$$\therefore \ x = -0.255 \text{ or } x = -11.7 \text{ [to 3 s.f.]}$$

Solve by completing the square

$$x^2 + 8x - 1 = 0$$

- Using the Quadratic Formula

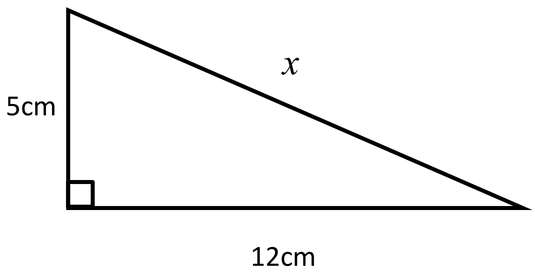
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$5x^2 + 2x - 3 = 3x^2 - 5x + 7$$
$$\Rightarrow 2x^2 + 7x - 10 = 0$$
$$[a = 2, b = 7, c = -10]$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-10)}}{2(2)}$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{129}}{4} \quad \text{[exact solutions]}$$

Solve By Using The Quadratic Formula

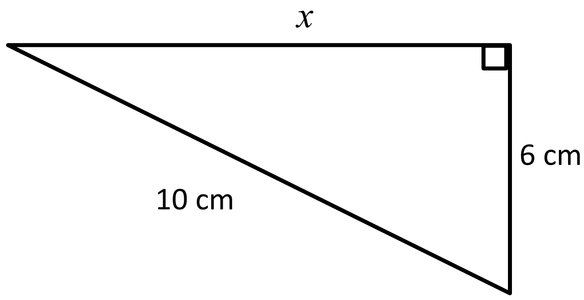
$$3x^2 - 5x - 2 = 0$$

Pythagoras



$$x^2 = 5^2 + 12^2$$
$$x^2 = 25 + 144$$
$$x^2 = 169$$
$$x = 13$$

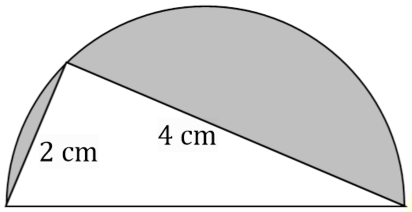
Find the value of x



Extension 1:

The diagram shows a triangle in a semi-circle.

Work out the perimeter of the semi-circle giving your answer as an exact form involving $\sqrt{5}$ and π. (no calculator / decimals)

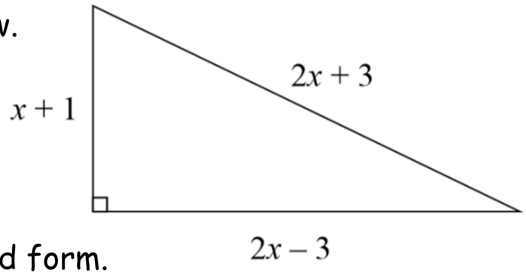


Extension 2:

Optional – have a go at these if you found the other sections straightforward.

If you're taking Further Maths you should definitely try this!

Find the area of the right-angled triangle below.



As an extra extension, leave your answer in surd form.

The following words or phrases are commonly used in A-level maths – how many do you recognise? Please jot down a brief definition of each term – look them up if you're not sure…

	Definition
Integer	
Rational number	
Numerator	
Denominator	
Irrational number	
Reciprocal	
Surd	
Polynomial	
Quadratic	
Linear	
Coefficient	
Gradient	
Parallel	
Perpendicular	
Tangent	
The Subject of an equation	